WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2020, held in 2021

MTMGDSE02T-MATHEMATICS (DSE1)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

   (a) State Parallelogram Law of forces.

   (b) If the resultant of two forces acting on a particle be at right angles to one of them, and its magnitude be one third of the magnitude of the other, then the ratio of the larger force to the smaller is $3 : 2\sqrt{2}$.

   (c) Define limiting friction.

   (d) If the masses of 3, 4, 5, 6 and 7 are placed at the four angular points A, B, C, D and the centre O respectively of a square of sides $2a$, then find the centre of gravity of the system.

   (e) State Lami's theorem.

   (f) What is astatic centre?

   (g) A particle describes a s.h.m. in a straight line with amplitude 2 cms. Its velocity while passing the centre of oscillation is 12 cm/sec. Find its time-period.

   (h) For a particle moving in a plane curve, if the tangential and normal accelerations are equal in magnitude then show that the velocity varies as e^{ψ}.

   (i) A particle moves in a straight line with an acceleration always directed towards a fixed point on it and proportional to its distance from it in a medium which offers a small resistance proportional to its velocity. Write down the equation of motion of the particle.

   (j) If the redial velocity of a particle is proportional to its cross-radial velocity, find the path in polar coordinates.

2. (a) Two forces of magnitudes $3P$, $2P$ respectively have resultant R. If the first force is doubled the magnitude of the resultant is doubled. Find the angle between the forces. 3

   (b) Three forces P, Q, R acting along $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ are in equilibrium. If O be the circumcentre of the triangle ABC, then prove that 5

$$\frac{P}{\frac{1}{b^2}+\frac{1}{c^2}-\frac{a^2}{b^2c^2}} = \frac{Q}{\frac{1}{c^2}+\frac{1}{a^2}-\frac{b^2}{c^2a^2}} = \frac{P}{\frac{1}{a^2}+\frac{1}{b^2}-\frac{c^2}{a^2b^2}},$$

   where a, b, c are the length of the sides BC, CA, AB respectively.

3. (a) A sphere of weight W is in equilibrium on a smooth plane of inclination α to the horizontal, being supported by a string, which is of length equal to the radius and is fastened to two points, one on the sphere and one on the plane. Prove that the tension of the string is $\frac{2}{3}\sqrt{3}W\sin\alpha$. 4

   (b) A heavy uniform beam rests with its extremities on two smooth inclined planes. The inclination of the inclined planes to the horizontal are α and β. Find the inclination of the beam with the horizontal in the position of equilibrium and the reaction of the inclined planes. 4

4. (a) Forces P, Q, R act respectively along the lines $x=0$, $y=0$ and $x\cos\alpha+y\sin\alpha=p$. Find the magnitude of the resultant and the equation of its line of action. 4

   (b) A solid hemisphere of weight W rests in limiting equilibrium with its curved surface on a rough inclined plane and the plane face is kept horizontal by a weight P attached to a point in the rim. Prove that the coefficient of friction is $P/\sqrt{W(2P+W)}$. 4

5. (a) If a system of coplanar forces be in equilibrium and each of the forces be rotated through the same angle, examine whether the system is also in equilibrium after the rotation. 4

   (b) Find the centre of gravity of a plate in the form of a quadrant AOB of a circle of radius a. 4

6. (a) Four rods are joined together to form a parallelogram, the opposite joints are joined by strings forming the diagonals and the whole system is placed on a smooth horizontal table. Show that their tensions are in the same ratio as their lengths. 5

   (b) Find the centre of gravity of the volume formed by revolving the area bounded by the parabolas $y^2=4ax$ and $x^2=4by$ about the axis of x. 3

7. (a) Prove that for a particle of mass m falling from rest under gravity from a height h above the ground, the sum of the K.E. and the P.E. of the particle is constant at every point of its path. 3

   (b) A uniform hemisphere of radius a and weight W, rests with its spherical surface on a horizontal plane and a rough particle of weight W' rests on the plane surface. Show that the distance of the particle from the centre of the plane face is not grater than $\frac{3W\mu a}{8W'}$, where μ is the coefficient of friction. 5

8. (a) A particle of mass m moves in a straight line under an attractive force mn^2x towards a fixed point on the line when at a distance x from it. If it be projected with a velocity V towards the centre of force from an initial distance a from it, prove that it reaches the centre of force after a time $\frac{1}{n}\tan^{-1}\frac{na}{V}$. 5

   (b) A particle moves with an acceleration which is always directed towards a fixed point O and equal to $\frac{\mu}{x}$, where x is the distance of the particle from O. If the particle starts from rest at a distance a from O then find the velocity of the particle when it is at a distance x from O. 3

9. (a) A point moves in a curve so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve. 3

   (b) Define S.H.M. Find the equation of motion of a particle of mass *m* executing S.H.M. of amplitude *a* and find the period of oscillation. 5

10.(a) A particle is acted on by a force parallel to the axis of *y* where acceleration is *ky* and is initially projected with a velocity $a\sqrt{k}$ parallel to the axis of *x* at a point where $y = a$. Prove that it will describe a catenary. 5

   (b) If a particle describe a rectangular hyperbola under a force which always parallel to an asymptotes. Find the law of force. 3

11.(a) Find the radial and cross radial components of velocity of a particle moving along a plane curve. 3

   (b) A straight smooth tube revolves with angular velocity Ω in a horizontal plane about one extremity which is fixed. If initially the particle is at rest at a distance *a* from the fixed end then find the distance of the particle from the fixed end at time *t*. 5

12. A particle of mass *m* is projected into the air with velocity in a direction making an angle α with the horizontal. To find the equations of motion and solve them to obtain the path described. 5+1+1+1

    Find also the time of flight taken by the particle to reach the horizontal plane again and the horizontal range covered.

    Also find the angle of projection for which the horizontal range is maximum.

13.(a) One end of an elastic string is fixed at *A* and the other end is fastened to a heavy particle, the modulus of elasticity of the string being equal to the weight of the particle. Show that if the particle is dropped from *A*, it will descend a distance $(2+\sqrt{3})\,a$ before coming to rest. 5

   (b) A particle *m* is attached by a light string, of length *l*, to a fixed point and oscillates under gravity through a small angle; find the period of its motion. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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