



WEST BENGAL STATE UNIVERSITY
B.Sc. Programme 5th Semester Examination, 2021-22

MTMGDSE02T-MATHEMATICS (DSE1)

MECHANICS

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any *five* questions from the following: 2×5 = 10
- (a) Write down the conditions of equilibrium of a system of coplanar forces acting on a rigid body.
- (b) Three forces P , Q , R act in the same sense along the sides \overline{BC} , \overline{CA} , \overline{AB} of a triangle ABC . If their resultant passes through the in-centre then show that $P + Q + R = 0$.
- (c) Find the centre of gravity of the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.
- (d) A heavy body is in limiting equilibrium on a rough inclined plane under the action of gravity only, then what is the inclination of the plane with the horizontal?
- (e) A particle moves along a straight line according to the law $s^2 = at^2 + bt + c$.
Prove that its acceleration varies as $\frac{1}{s^3}$.
- (f) At what height would the kinetic energy of a falling particle be equal to half of its potential energy?
- (g) If a particle moves in a circle of radius r with uniform speed v , then find its angular velocity about the centre.
- (h) A particle is projected under gravity at an angle α with the horizontal. Find the velocity of the particle at time t .
- (i) A particle describes the curve $r = ae^\theta$ with constant angular velocity. Show that its radial acceleration is zero and the transverse acceleration varies as the distance from the pole.
2. Three forces P , Q , R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the equation of the line of action of their resultant. 8

3. Show that the least force which will move a weight W along a rough horizontal plane is $W \sin \phi$, where ϕ is the angle of friction. 8
4. A frustum of a cone is formed by cutting off the upper portion of a solid right circular cone by a plane parallel to the base. The radii of the parallel circular sections being R and r , and h the height of the frustum, show that the height of the centre of gravity of the frustum from the base is $\frac{h}{4} \cdot \frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}$. 8
5. (a) A cycloid is placed with its axis vertical and vertex downwards. Show that a particle cannot rest at any point of the curve which is higher than $2a \sin^2 \lambda$ above the lowest point, where λ is the angle of friction and a is the radius of the generating circle of the cycloid. 3
- (b) Two equal uniform rods AB and AC , each of length $2b$ are freely jointed at A and rest on a smooth vertical circle of radius a . Show that if 2θ be the angle between them, then $b \sin^3 \theta = a \cos \theta$. 5
6. (a) Deduce the expressions for tangential and normal components of the acceleration of a particle describing a plane curve. 5
- (b) A particle describes a circle of radius a in such a way that its tangential acceleration is K times the normal acceleration, where K is a constant. If the speed of particle at any point be u , prove that it will return to the same point after a time
$$\frac{a}{Ku} (1 - e^{-2\pi K})$$
 3
7. Two particles are projected simultaneously from O in different directions with same speed u so as to pass through another point P . If α and β are the angles of projection prove that they pass through P at times separated by
$$\frac{2u}{g} \sin \frac{1}{2}(\alpha - \beta) \cdot \sec \frac{1}{2}(\alpha + \beta)$$
 8
8. (a) A particle of mass m falls from rest at a distance a from the centre of force varying inversely as the square of the distance from the centre. Find the time it descends to the centre of force. 4
- (b) A particle moving in a straight line starts from rest and the acceleration at any time t is $a - Kt^2$, where a and K are positive constant. Show that the maximum velocity attained by the particle is $\frac{2}{3} \sqrt{\frac{a^3}{K}}$. 4
9. (a) A particle rests in equilibrium under the attraction of two centre of force which attract directly as the distance, their attractions at unit distance being μ_1 and μ_2 respectively. The particle is slightly displaced towards one of the centres, show that the time of small oscillation is
$$\frac{2\pi}{\sqrt{\mu_1 + \mu_2}}$$
. 4

- (b) In a simple harmonic motion, if f be the acceleration and v be the velocity at any instant and T is periodic time, then show that $f^2 T^4 + 4\pi^2 v^2 T^2 = 16\pi^4 a^2$. 4
- 10.(a) A particle is projected vertically upwards with a velocity u in a medium whose resistance varies as the square of the velocity. Investigate the motion. 4
- (b) If the radial and transverse velocities of a particle are $\mu\theta$ and λr respectively, show that the path of the particle can be represented by an equation of the form $r = A\theta^2 + B$. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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